The Bramble Academy Calculation Policy

September 2022/2023

Date of policy: 16th September 2022

Member of staff responsible: Natalie Aveyard

Review Date: July 2023

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General Data Protection Regulation

This policy has been reviewed in accordance with the General Data Protection Regulation (GDPR) which will replace the Data Protection Act 1998 from 25th May 2018. The introduction of GDPR has resulted in changes to many existing data protection rules and regulations that educational establishments adhere to. The Evolve Trust has undertaken a full data protection audit and have ensured that appropriate changes that have been made to data protection rules and regulations have been adhered to in full. The Evolve Trust has carried out all additional compliance requirements and fully accepts their duty of care to ensure individuals' data is kept safe and secure, resulting in increased compliance in our systems, processes and policies.

Aims of the policy

This policy is largely taken from the White Rose Maths calculation policy and supports the White Rose Maths scheme of learning that we use within our primaries. Furthermore, it complements the progression and expectations detailed within the National Curriculum Mathematics programmes of study for Key Stage 1 and 2 (see Appendix A). The aim is to ensure consistency and progression in our approach to calculation, making connections within mathematics, and enable a smooth transition between year groups and phases. Furthermore, to develop children who can calculate with accuracy, confidence and fluency both mentally and using a formal written method and apply those skills to a variety of contexts to solve problems and reason.

The policy includes the 4 operations of addition, subtraction, multiplication and division and uses a Concrete-Pictorial-Abstract (CPA) approach to learning.

> **Concrete** – children should have the opportunity to use concrete objects and manipulatives to help them understand what they are doing.
>
> **Pictorial** – alongside this children should use pictorial representation. These representations can then be used to help reason and solve problems.
>
> **Abstract** – both concrete and pictorial representations should support children's understanding and abstract methods.

At Evolve we believe that when children are introduced to a new concept, they should have the opportunity to develop a deep understanding of that concept so that it becomes lifelong learning.

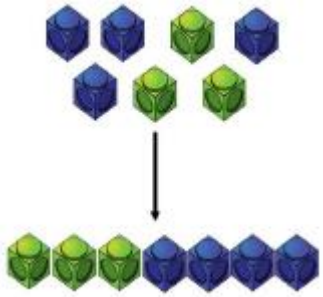
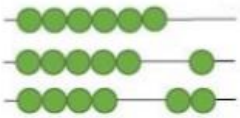
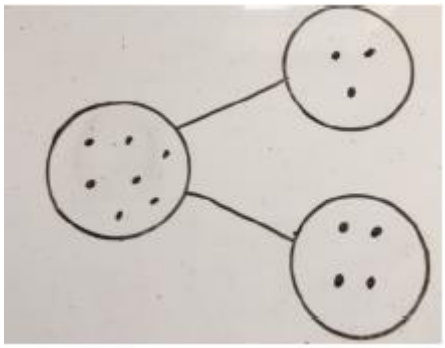
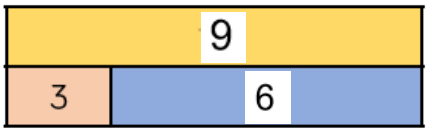
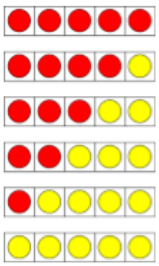
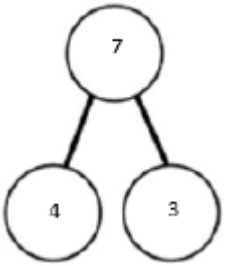
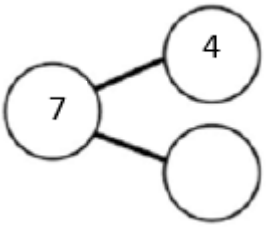
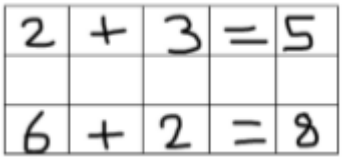
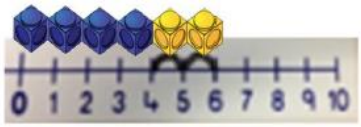
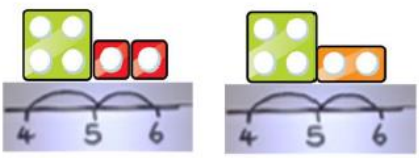
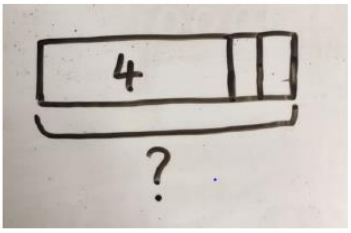
Guidance

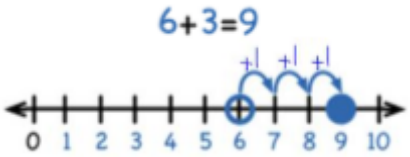
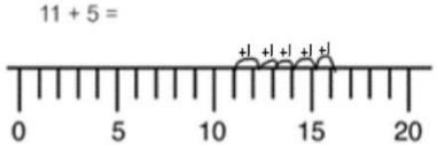
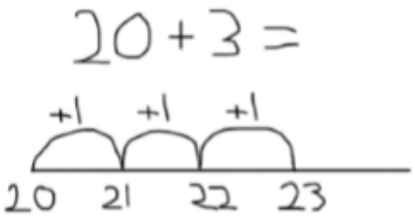
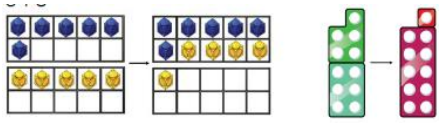
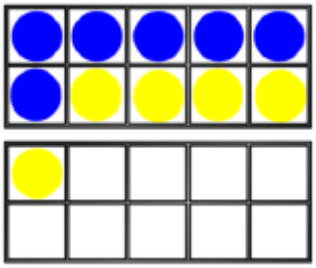
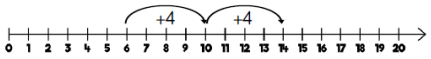
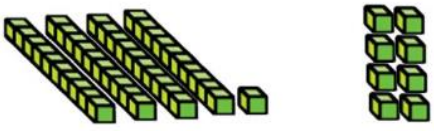
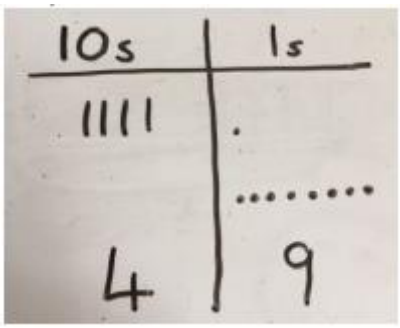
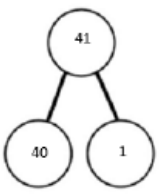
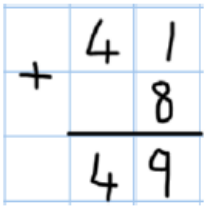
	EYFS / Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Combining two parts to make a whole: part whole model. Starting at the bigger number and counting on using cubes. Regrouping to make 10 using ten frame.	Adding three single digits. Use of base 10 to combine two numbers.	Column method regrouping. Using place value counters (up to 3 digits).	Column method – regrouping (up to 4 digits)	Column method – regrouping. Use of place value counters for adding decimals.	Column method – regrouping. Abstract methods. Place value counters to be used for adding decimal numbers.
Subtraction	Taking away ones. Counting back. Find the difference. Part-whole model. Make 10 using the 10 frame.	Counting back. Find the difference. Part-whole model. Make 10. Use of base 10.	Column method with regrouping. (up to 3 digits using place value counters).	Column method with regrouping. (up to 4 digits).	Column method with regrouping. Abstract for whole numbers. Start with place value counters for decimals – with the same amount of decimal places.	Column method with regrouping. Abstract methods. Place value counters for decimals – with different amounts of decimal places.

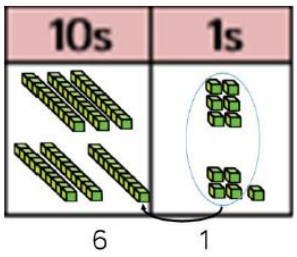
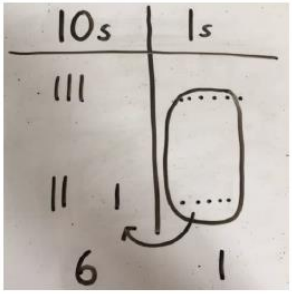
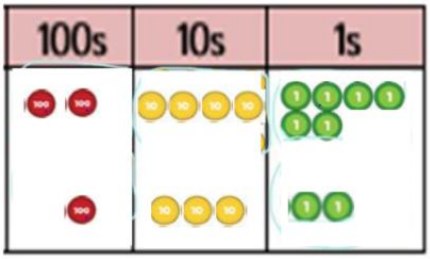
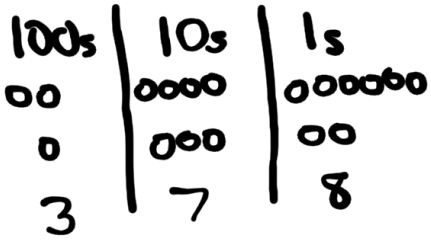
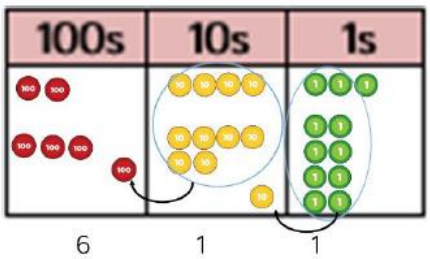
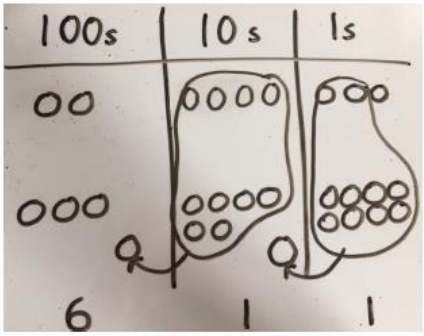
Multiplication	Recognising and making equal groups. Doubling. Counting in multiples; using cubes, Numicon and other objects in the classroom.	Arrays – showing commutative multiplication.	Arrays. 2d x 1d using base 10.	Column multiplication – introduced with place value counters. (2 and 3 digit multiplied by 1 digit).	Column multiplication. Abstract only but might need a repeat of year 4 first (up to 4 digit numbers multiplied by 1 or 2 digits).	Column multiplication. Abstract methods (multi-digit up to 4 digits by a 2 digit number).
	EYFS / Year 1	**Year 2**	**Year 3**	**Year 4**	**Year 5**	**Year 6**
Division	Sharing objects into groups. Division as grouping e.g. I have 12 sweets and put them in groups of 3, how many groups? Use cubes and draw round 3 cubes at a time.	Division as grouping. Division within arrays – linking to multiplication. Repeated subtraction.	Division with a remainder – using lollipop sticks, times tables facts and repeated subtraction. 2d divided by 1d using base 10 or place value counters.	Division with a remainder. Short division (up to 3 digits by 1 digit- concrete and pictorial).	Short division. (up to 4 digits by a 1 digit number including remainders).	Short division. Long division with place value counters (up to 4 digits by a 2 digit number). Children should exchange into the tenths and hundredths column too.

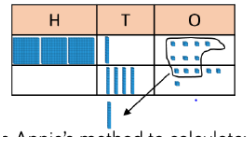
Refer to Appendix A for details from the DfE Mathematics programmes of study; key stages 1 and 2.

Addition

Concrete	Pictorial	Abstract
<u>Combining two parts to make a whole</u> (use other resources too e.g. counters, shells, teddy bears, cars).  **Using bead strings:**  $6 = 6 + 0$ $6 = 5 + 1$ $6 = 4 + 2$	**Children to represent the cubes using dots or crosses. They could put each part on a part whole model too.**  **or represent using a bar model.**  **Bridging from a pictorial to abstract;**  $5 = 5 + 0$ $5 = 4 + 1$ $\_ = \_ + \_$ $\_ = \_ + \_$ $\_ = \_ + \_$ $\_ = \_ + \_$	**4 + 3 = 7** **Four is a part, 3 is a part and the whole is seven.**  **Varied fluency:**  **7 = 4 + ◯** **◯ + 4 = 7** **As children start to write number sentences into their maths books (squared maths paper) ensure one digit in each box and a line between each question so that students can go back and correct if necessary.** 
<u>Counting on using number lines</u> using cubes or Numicon   	**A bar model which encourages the children to count on, rather than count all.** **4 + 2 = 6**  **Children draw their own number line**	**The abstract number line:** **What is 2 more then 4?** **What is the sum of 2 and 4?** **What is the total of 4 and 2?** **4 + 2**  **Partial number lines can be used as a transition to open number lines**

		 **Using an open number line** 
<u>Regrouping to make 10;</u> using ten frames and counters/cubes or using Numicon. **6 + 5** 	**Children to draw the ten frame and counters / cubes.**  **Using a number line:** **6 + 8 becomes 6 + 4 + 4…the 8 is partitioned to 4 + 4** 	**Children to develop an understanding of equality e.g.** $6 + \square = 11$ $6 + 5 = 5 + \square$ $6 + 5 = \square + 4$
Please Note: As we move forward into more formal methods and developing an understanding of what is happening, the following must be stressed: **1) The units must be added first!** **2) 'Carry' numbers underneath the bottom line!** **3) Reinforce the place value! It is not 6 add 8, it is 6 tens add 8 tens!**		
<u>TO + O using base 10.</u> Continue to develop understanding of partitioning and place value. **41 + 8** 	**Children to represent the base 10 e.g. lines for tens and dot/crosses for ones.** 	**41 + 8**  1 + 8 = 9 40 + 9 = 49 
<u>TO + TO using base 10.</u> Continue to develop understanding of	**Children to represent the base 10 in a place value chart.**	**Looking for ways to make 10.**

partitioning and place value. **36 + 25**  	 	36 + 25= 1 5 30 + 20 = 50 5 + 5 = 10 50 + 10 + 1 = 61 Formal method: 36 +25 61 1
<u>Use of place value counters to add HTO to TO, HTO to HTO without carrying</u> **246 + 132**  **3 7 8** **Or base 10 equipment could be used**	**Children to represent the place value counters in a place value chart**  	246 + 132 = 378 200 + 40 + 6 <u>100 + 30 + 2</u> 300 + 70 + 8 = 378 **and** 343 <u>+ 116</u> 400 50 <u>+ 9</u> 459 **To** 116 + 343= 459 343 <u>+ 116</u> 459
<u>Use of place value counters to add HTO + TO, HTO + HTO etc. with carrying</u> When there are 10 ones in the 1s column – we exchange for 1 ten, when there are 10 tens in the 10s column – we exchange for 1 hundred.  **Or base 10 could be used**	**Children to represent the counters in a place value chart, circling when they make an exchange.**  	243 +368 611 1 1

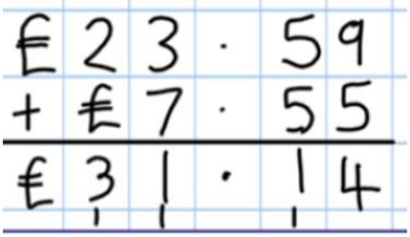


	3	1	7
+		4	6
	3	6	3
		1	

In Year 4 the above is extended to adding with numbers up to 4 digits. Children will consolidate their use of the traditional column method and will be able to use it confidently to add numbers up to 4 digits. This could include carrying units, tens and hundreds.

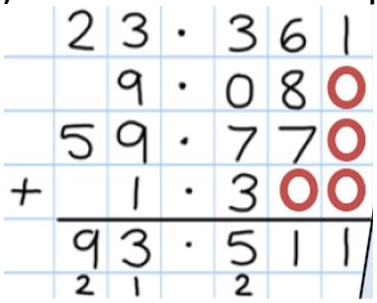
In year 5 children will now use the column method to extend to more than 4 digits and solving problems with increasing complexity; multi-step in context. They will also add decimal numbers in the context of money and measures. It is important that children have place value skills beyond 4 digits here and fully understand what a decimal number represents.
As previously, place value counters can be used to support the concrete representation and children can use a pictorial representation to support their independent calculations if needed.
Some additional points to support with the progression into working with decimals:

1). The decimal point needs to be lined up just like all of the other place value columns and must be remembered in the final answer.



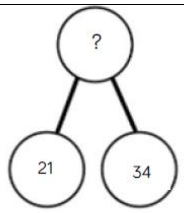
2). It is important that children say 3 tenths add 4 tenths instead of 3 add 4 to show an understanding of the value of the digits ie 0.3 + 0.4…they are not adding whole numbers.

3). Zeros should be added to support place value, showing that there is no value to add.



In year 6 children need to use all the previous adding skills developed to add several numbers with a variety of different decimal places. Many of these problems will be in the context of money or measures.

Conceptual variation; difference ways to ask children to solve 21 + 34



?	
21	34

Word problems:
In year 3, there are 21 children and in year 4, there are 34 children. How many children in total?

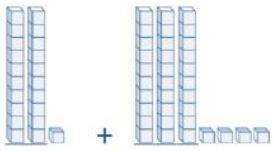
21 + 34 = 55. Prove it.

21
+34
___

21 + 34 =

☐ = 21 + 34

Calculate the sum of twenty-one and thirty-four.

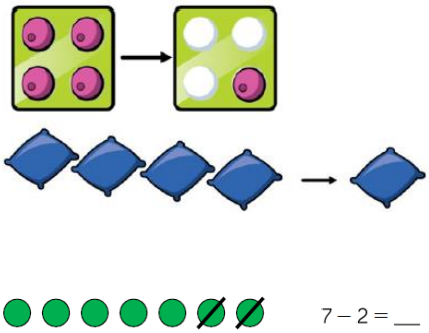
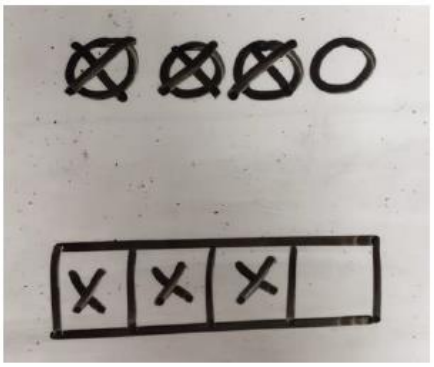
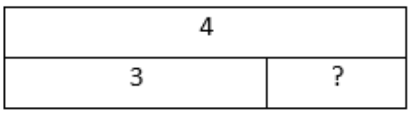
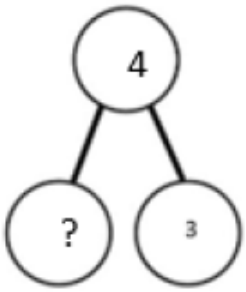
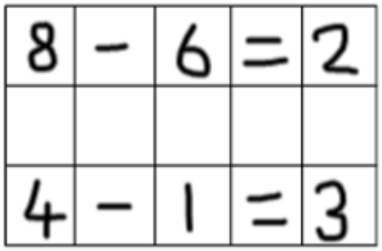
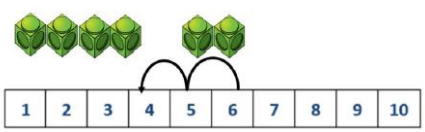
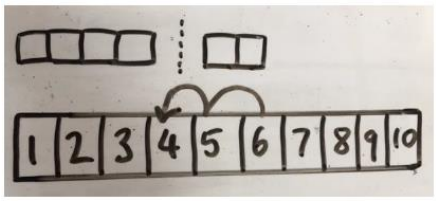
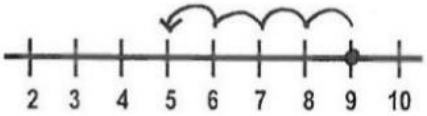
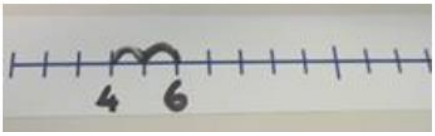


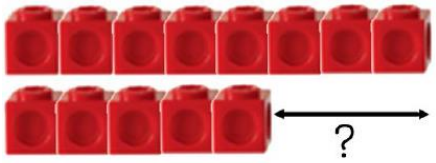
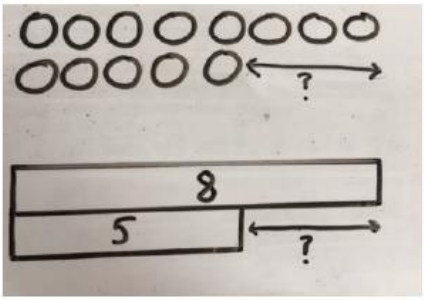
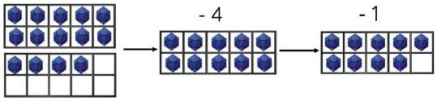
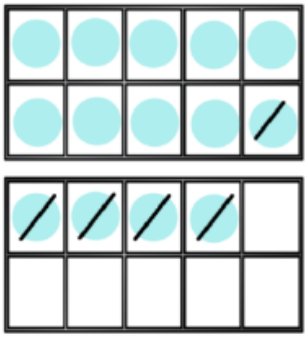
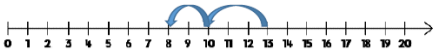
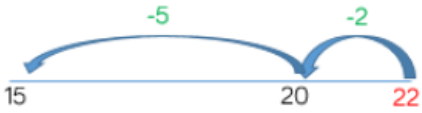
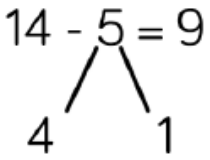
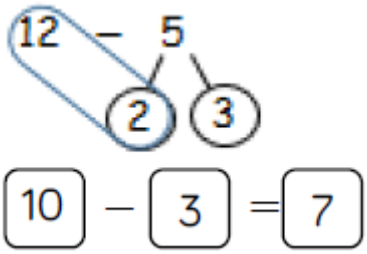
Missing digit problems:

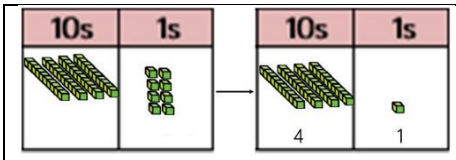
10s	1s
10 10	1
10 10 10	?
?	5

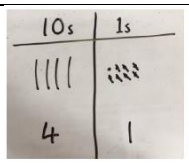
Key vocabulary: Add, more than, plus, make, altogether, equals, most, count on, addition, total, sum, increase.

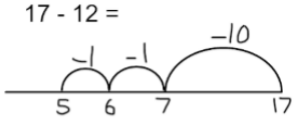
Subtraction

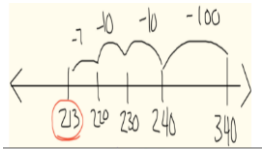
Concrete	Pictorial	Abstract
<u>Physically taking away and removing objects from a whole</u> (ten frames, Numicon, cubes, counters and other items such as beanbags could be used). **4 – 3 = 1**  	**Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.** 	4- 3 = ☐ = 4 - 3   **As children start to write number sentences into their maths books (squared maths paper) ensure one digit in each box and a line between each question so that students can go back and correct if necessary.** 
<u>Counting back</u> (using number lines or number tracks) children start with 6 and count back 2. 6 - 2 = 4  	**Children to represent what they see pictorially e.g.**  **Children draw their own number line:** 9 - 4 = 5 -1 -1 -1 -1  	**Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line.**  

<u>Finding the difference</u> (using cubes, Numicon or Cuisenaire rods, other objects can also be used). **Calculate the difference between 8 and 5.**  	**Children to draw the cubes / other concrete objects which they have used or use the bar model to illustrate what they need to calculate.**  	**Find the difference between 8 and 5.** **8 – 5, the difference is ◯** **Children to explore why** **9 – 6 = 8 – 5 = 7 – 4 have the same difference.**
<u>Making 10</u> using ten frames. **14 – 5**  	**Children to present the ten frames pictorially and discuss what they did to make 10.**  **Or children can draw their own number line:** **13 – 5 (13 – 3 - 2)**  **Progressing to an open number line:** **22 - 7**  	**Children to show how they can make 10 by partitioning the subtrahend.**  14 – 4 = 10 10 – 1 = 9 
Please Note: As we move forward into more formal methods and developing an understanding of what is happening, the following must be stressed: **1) The units must be subtracted first!** **2) 'Carry' numbers underneath the bottom line!** **3) Reinforce the place value! It is not 8 subtract 6, it is 8 tens subtract 6 tens!**		
<u>Column method</u> using base 10 NO exchanging. **48 – 7**	**Children to represent the base 10 pictorially.**	**Column method or children could count back 7.**

<table>
<tr>
<td> 

Or using place value counters.</td>
<td> 

Varied fluency - by partitioning numbers:
34 – 13 = ____
34
30 4
–10 –3
20 1
Using a number line:
17 - 12 =
 

340 - 127
 
</td>
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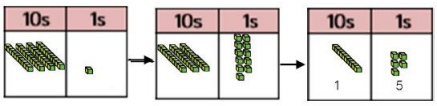
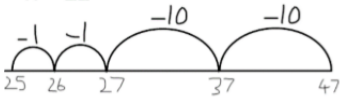
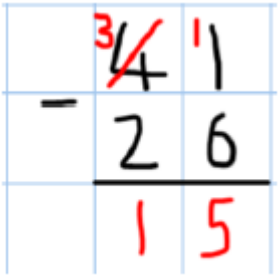
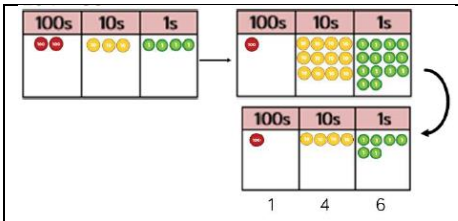
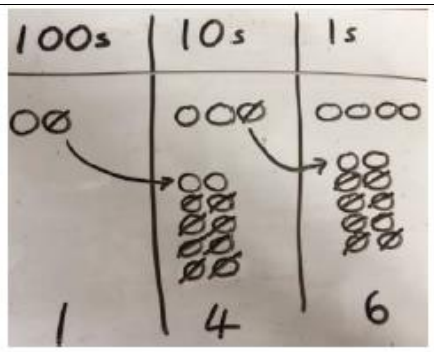
</tr>
<tr>
<td><u>Column method</u> using base 10 and having to exchange.
41 – 26
 

Or using place value counters</td>
<td>Represent the base 10 pictorially, remembering to show the exchange.
 

Varied fluency - by partitioning numbers:
42 – 15 =
42
40 2
–10 –5
We can't subtract the ones. Can we partition differently?
42
30 12
–10 –5
20 7
Now we can subtract the ones and then subtract the tens.
42 – 15 = 27
Using a number line:
47 - 22 =
 
</td>
<td>Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because 41 = 30 + 11
 
</td>
</tr>
<tr>
<td><u>Column method</u> using place value counters and having to exchange.
234 – 88</td>
<td>Represent the place value counters pictorially; remembering to show what has been exchanged.</td>
<td>Formal column method. Children must understand what has</td>
</tr>
</table>

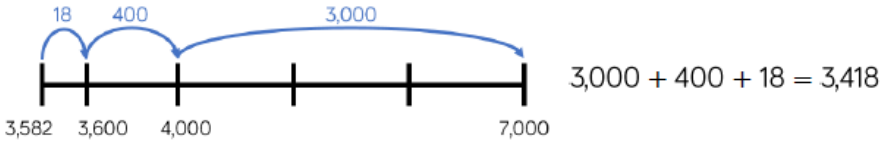




happened when they have crossed out digits.

$$\begin{array}{r} {}^{2}2\,{}^{1}3\not{4} \\ -\ 88 \\ \hline 6 \end{array}$$

In Year 4 the above is extended to subtracting with numbers up to 4 digits. Children will consolidate their use of the traditional column method and will be able to use it confidently to subtract numbers up to 4 digits. This could include making more than one exchange and looking at efficient mental methods.
e.g. 7000 – 3582 – making connections with addition:



Or subtracting with by partitioning ie 7000 – 3000 = 4000, 4000 – 500 = 3500, 3500 – 80 = 3420, 3420 – 2 = 3418

In year 5 children will now use the column method to extend to more than 4 digits and solving problems with increasing complexity; multi-step in context. They will also subtract decimal numbers in the context of money and measures. It is important that children have place value skills beyond 4 digits here and fully understand what a decimal number represents.
As previously, place value counters can be used to support the concrete representation and children can use a pictorial representation to support their independent calculations if needed.
Some additional points to support with the progression into working with decimals:

1). The decimal point needs to be lined up just like all of the other place value columns and must be remembered in the final answer.

$$\begin{array}{r} {}^{6}\not{7}\,{}^{10}\not{X}\,{}^{1}6\,{}^{8}\not{9}\cdot{}^{1}0 \\ -\quad 3\,7\,2\cdot 5 \\ \hline 6\,7\,9\,6\cdot 5 \end{array}$$

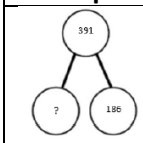
2). It is important that children say 4 tenths subtract 3 tenths instead of 4 subtract 3 to show an understanding of the value of the digits ie 0.4 - 0.3…they are not adding whole numbers.

3). Zeros should be added to support place value, showing that there is no value to add.

$$\begin{array}{r} {}^{9}\not{X}\,{}^{10}\not{0}\,{}^{1}5\cdot{}^{3}\not{4}\,1\,9 \\ -\quad 3\,6\cdot 0\,8\,0 \\ \hline 6\,9\cdot 3\,3\,9 \end{array}$$

In year 6, children need to use mental methods and the compact column method of subtraction to solve an increasingly complex range of calculation including those with integers, those with decimals and those with mixed numbers.

Conceptual variation; different ways to ask children to solve 391 - 186



391	
186	?

Raj spent £391, Timmy spent £186. How much more did Raj spend?

Calculate the difference between 391 and 186.

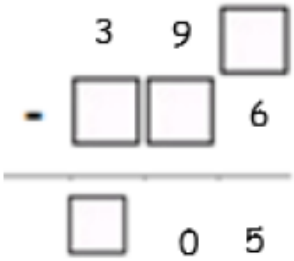
□ = 391 – 186

391

-186

What is 186 less than 391?

Missing digit calculations.



Key vocabulary: take, take away, less than, the difference, difference between, subtract, minus, fewer, decrease, how many left.

Multiplication

Multiplication tables

At Evolve we believe that fluency in multiplication tables is an essential ingredient for future success in mathematics. To support the development of this skill with our children we implement the following approach:

Year 1 – Counting in 2s, 5s and 10s.

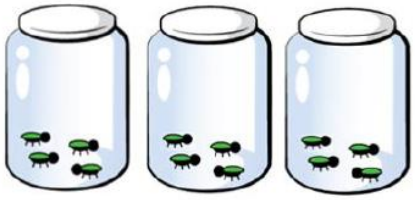
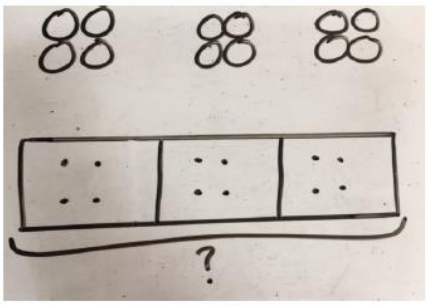
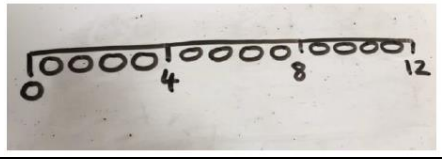
Year 2 – Know their 2, 5 and 10 multiplication tables.

Year 3 – Know their 3, 4 and 8 multiplication tables.

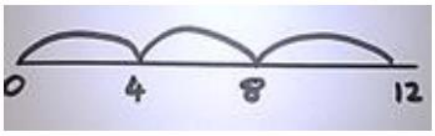
Year 4 – Know their 6, 7, 9, 11 and 12 multiplication tables.

Year 5 and 6 – Continued practise to develop fluency and application to wider problem solving in a context.

It is paramount that students are given opportunity to practise their times tables on a daily basis, whether as an explicit activity or integrated within the main part of the lesson. In addition to this children should be encouraged to complete home study on this several times a week.

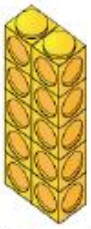
Concrete	Pictorial	Abstract
<u>Repeated grouping / repeated addition</u> **3 x 4** **4 + 4 + 4** **There are 3 equal groups, with 4 in each group.**   **Using Numicon** 	**Children to represent the practical resources in a picture and use a bar model.** 	**3 x 4 = 12** **4 + 4 + 4 = 12**
<u>Number lines to show repeated groups</u> – **3 x 4**	**Represent this pictorially alongside a number line e.g.** 	**Abstract number line showing three jumps of four.** **3 x 4 = 12**

Cuisenaire rods can be used too.

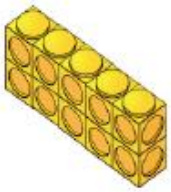


<u>Use arrays to illustrate commutativity</u> counters and other objects can also be used.

2 x 5 = 5 x 2

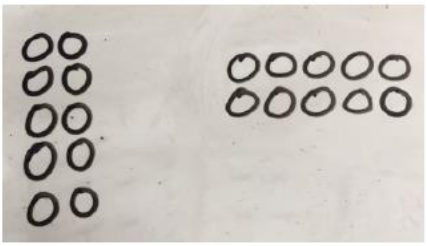


2 lots of 5



5 lots of 2

Children to represent the arrays pictorially.



Children to be able to use an array to write a range of calculations e.g.

10 = 2 x 5
5 x 2 = 10
2 + 2 + 2 + 2 + 2 = 10
10 = 5 + 5

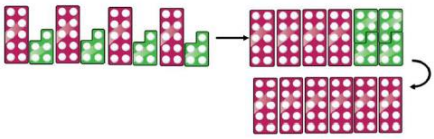
Bar model:
7 x 5 or 5 x 7

35						
5	5	5	5	5	5	5

35				
7	7	7	7	7

<u>Partition to multiply</u> using Numicon, base 10 or Cuisenaire rods.

4 x 15



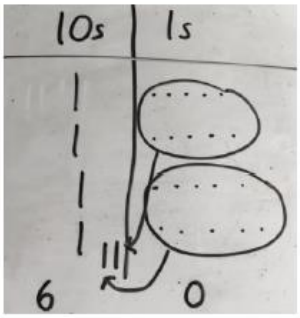
21 x 3

Tens	Ones

22 x 4

Tens	Ones
10 10	1 1
10 10	1 1
10 10	1 1
10 10	1 1

Children to represent the concrete manipulatives pictorially.

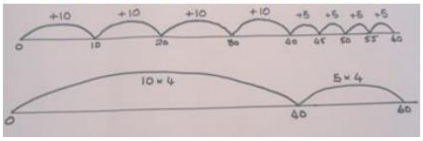


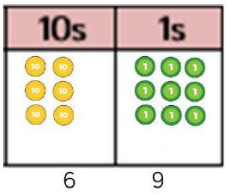
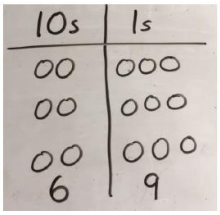
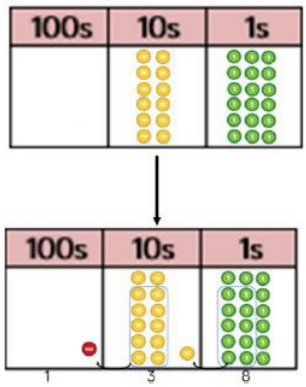
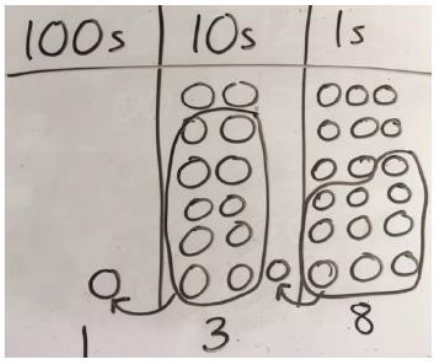
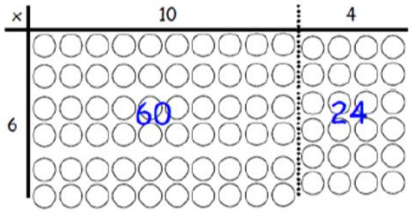
Children to be encouraged to show the steps they have taken.

4 x 15
10 5

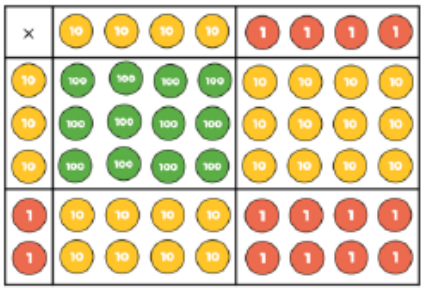
10 x 4 = 40
5 x 4 = 20
40 + 20 = 60

A number line can also be used

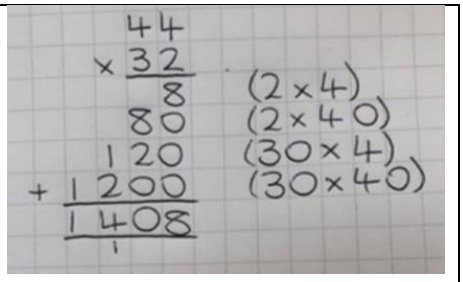


<u>Formal column method</u> with place value counters (base 10 can also be used). **3 x 23** 	**Children to represent the counters pictorially.** 	**Children to record what it is they are doing to show understanding.** 3×23 $20 \quad 3$ $3 \times 20 = 60$ $3 \times 3 = 9$ $60 + 9 = 69$ $\begin{array}{r} 23 \\ \times\ 3 \\ \hline 69 \end{array}$
<u>Formal column method</u> with place value counters. Extend to TO x O, HTO x O, ThHTO x O.. **6 x 23** 	**Children to represent the counters / base 10, pictorially e.g. the image below.**  **Other representations students could use to support:** **6 x 14**  **7 x 35** | × | 30 | 5 | | 7 | 210 | 35 | **210 + 35 = 245**	**Formal written method.** **6 x 23** **23** **<u>x 6</u>** **18 (6 x 3)** **+<u>120</u> (6 x 20)** **138** **leading to….** $6 \times 23 =$ $\begin{array}{r} 23 \\ \times\ 6 \\ \hline 138 \\ \hline 1\ 1 \end{array}$

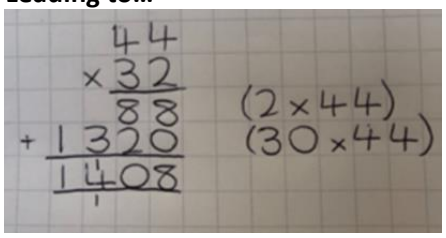
Formal column method TO x TO
with place value counters
44 x 32



×	40	4
30	1,200	120
2	80	8

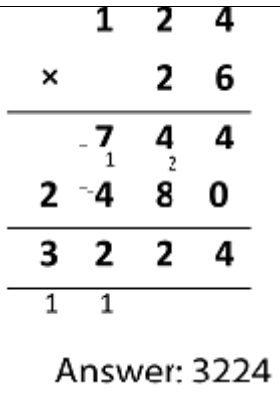


Leading to…

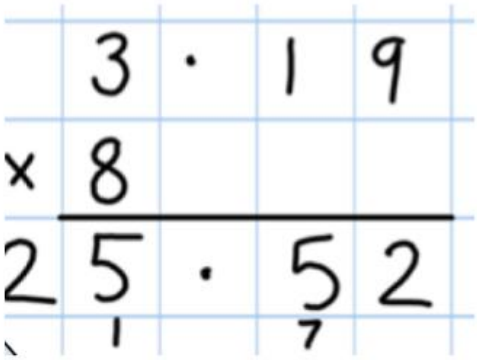


When children start to multiply 3d x 3d and 4d x 2d etc., they should be confident with the abstract:

To get 744 children have solved 6 x 124
To get 2480 they have solved 20 x 124

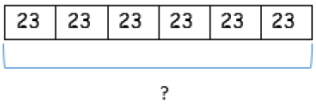


In year 6 children will consolidating short and long multiplication and they will also learn the new skill of using short multiplication to multiply decimal numbers to 2 decimal places by 1 digit.



When multiplying decimals it is important to remember that the digit you are multiplying by needs to be lined up with the ones digits. As with all decimal work, the decimal points must be lined up and the children need to have a clear understanding why that is.

Conceptual variation; different ways to ask children to solve 6 x 23



Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week?

With the counters, prove that 6 x 23 = 138.

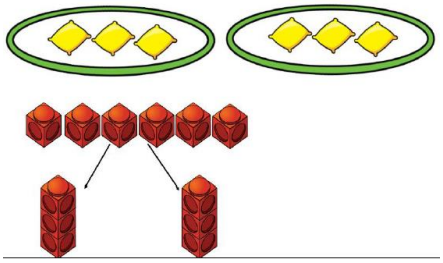
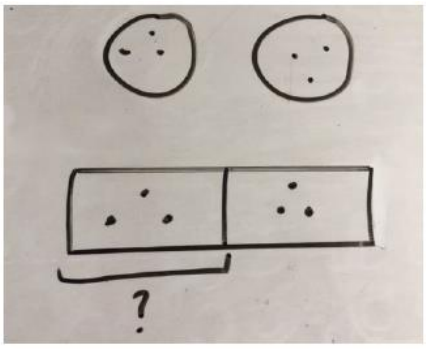
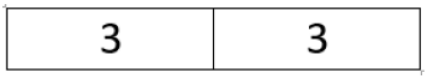
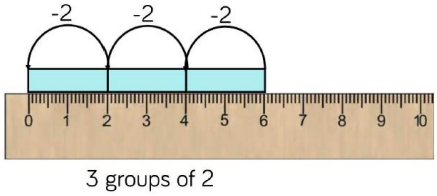
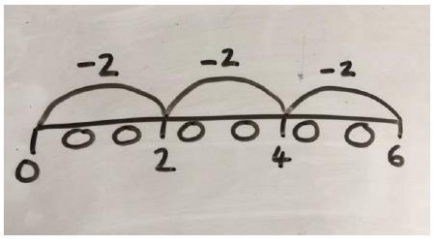
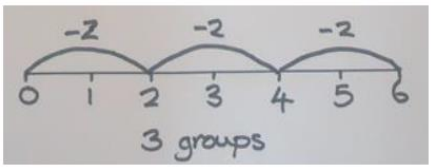
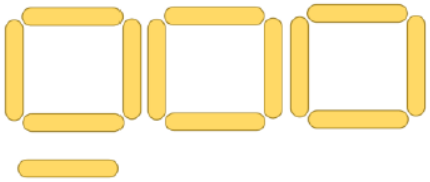
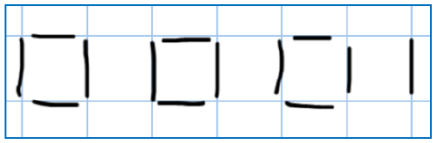
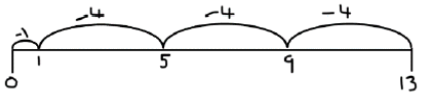
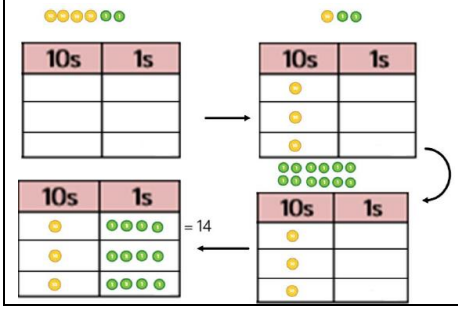
Find the product of 6 and 23

What is the calculation?
What is the product?

		$6 \times 23 =$ $\square = 6 \times 23$ 6×23 $\quad$ 23×6	

Key vocabulary: double, times, multiply, multiplied by, repeated addition, the product of, groups of, lots of, equal groups, commutative, sets of.

Division

Concrete	Pictorial	Abstract
<u>Sharing</u> using a range of objects. **6 ÷ 2**  **The idea of EQUAL GROUPS needs to be stressed.**	**Represent the sharing pictorially.** 	**6 ÷ 2 = 3**  **Children should also be encouraged to use the times tables facts to support, understanding that multiplication and division are inverses.**
<u>Repeated subtraction</u> using Cuisenaire rods above a ruler. **6 ÷ 2** 	**Children to represent repeated subtraction pictorially.** 	**Abstract number line to represent the equal groups that have been subtracted.** 
<u>2d + 1d with remainders</u> using lollipop sticks. Cuisenaire rods, above a ruler can also be used. **13 ÷ 4** **Use of lollipop sticks to form wholes – squares made because we are dividing by 4**  **There are 3 whole squares, with 1 left over.**	**Children to represent the lollipop sticks pictorially** **There are 3 whole squares with 1 left over.** 	**13 ÷ 4 = 3 remainder 1** **Children should be encouraged to use their times table facts; they could also present repeated addition on a number lin.** **'3 groups of 4, with 1 left over.'** 
<u>Sharing using place value counters.</u> **42 ÷ 3 = 14** 	**Children to represent the place value counters pictorially.**	**Children to be able to make sense of the place value counters and write calculations to show the process** **42 ÷ 3** **42 = 30 + 12** **30 ÷ 3 = 10** **12 ÷ 3 = 4** **10 + 4 = 14**

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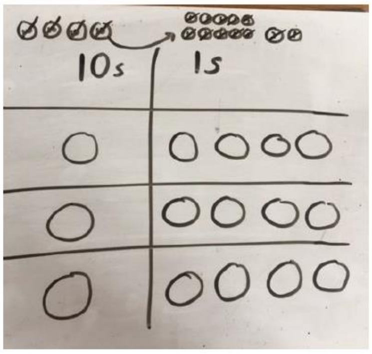

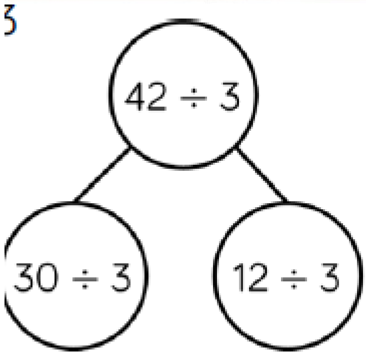

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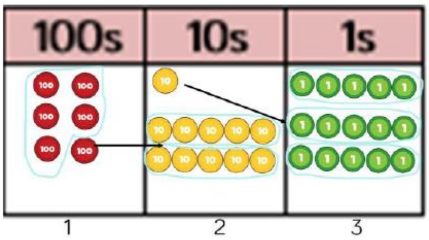
<u>Short division</u> using place value counters to group.
615 ÷ 5



1. **Make 615 with place value counters.**
2. **How many groups of 5 hundreds can you make with 6 hundred counters?**
3. **Exchange 1 hundred for 10 tens.**
4. **How many groups of 5 tens can you maek with 11 ten counters?**
5. **Exchange 1 ten for 10 ones.**
6. **How many groups of 5 ones can you make with 15 ones?**

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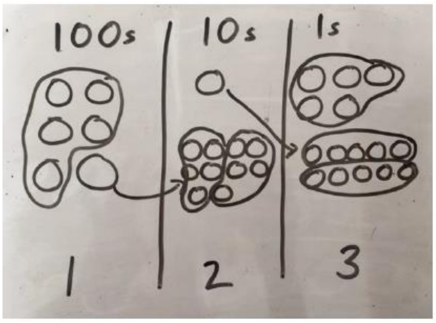
Represent the place value counters pictorially.



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Children to write the calculation using the short division scaffold.

$$\begin{array}{r} 123 \\ 5\,\overline{\smash{)}\,6\,{}^{1}1\,{}^{1}5} \end{array}$$

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<u>Short division</u> with remainders using place value counters to group.
4894 ÷ 4

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Represent the place value counters pictorially.

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Children to write the calculation using the short division scaffold.

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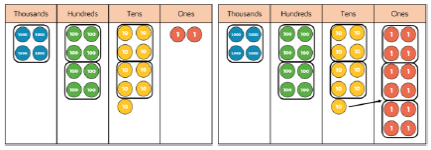
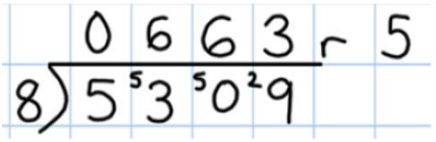


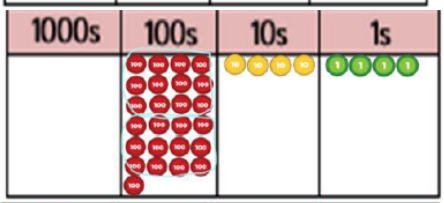
	1	2	2	3
4	4	8	9	12

5309 ÷ 8



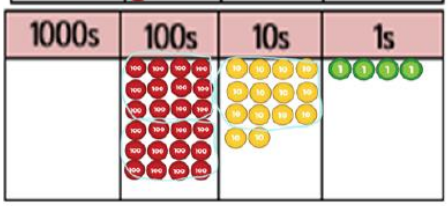
<u>Long division</u> using place value counters.
2544 ÷ 12

We can't group 2 thousands into groups of 12 so will exchange them.



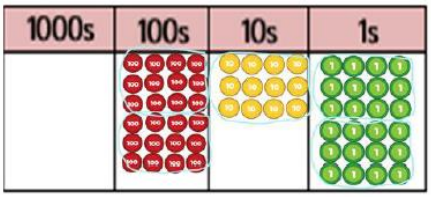
We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

$$\begin{array}{r} 02 \\ 12\overline{)2544} \\ 24 \\ \hline 1 \end{array}$$



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

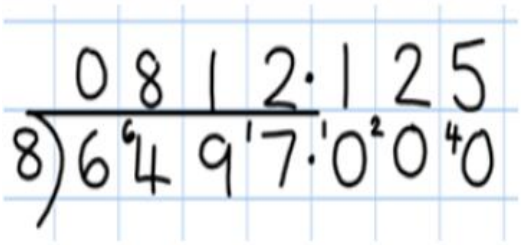
$$\begin{array}{r} 021 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 2 \end{array}$$



After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.

$$\begin{array}{r} 0212 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 24 \\ 24 \\ \hline 0 \end{array}$$

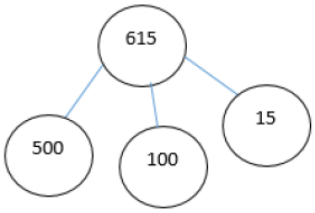
In Year 6 children also look at how the remainders are expressed- children need to express remainders as decimals and fractions- depending on the context of the question.



The remainder in this answer would have been 1 but it has been expressed as a decimal. To do this, children need to insert a decimal point next to the units and carry the remainder over the decimal point. Zeroes are inserted to the right of the decimal point to show that there was no value.

Conceptual variation; different ways to ask children to solve 615 ÷ 5

Using the part whole model below, how can you divide 615 by 5 without using short division?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

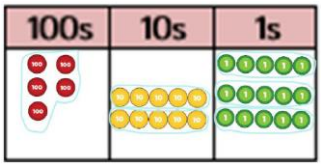
615 pupils need to be put into 5 groups. How many will be in each group?

$5\overline{)615}$

$615 \div 5 =$

$\square = 615 \div 5$

What is the calculation? What is the answer?



Key vocabulary: share, share equally group, divide, divided by, half, groups of, lots of, array, divided into, division, grouping, left, left over, short division, carry, remainder, divisible by, factor, quotient,

Appendix A: Mathematics programme of study: key stages 1 and 2

Year 1

Number – addition and subtraction

Statutory requirements

Pupils should be taught to:

- read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs
- represent and use number bonds and related subtraction facts within 20
- add and subtract one-digit and two-digit numbers to 20, including zero
- solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = ☐ – 9.

Notes and guidance (non-statutory)

Pupils memorise and reason with number bonds to 10 and 20 in several forms (for example, 9 + 7 = 16; 16 – 7 = 9; 7 = 16 – 9). They should realise the effect of adding or subtracting zero. This establishes addition and subtraction as related operations.

Pupils combine and increase numbers, counting forwards and backwards.

They discuss and solve problems in familiar practical contexts, including using quantities. Problems should include the terms: put together, add, altogether, total, take away, distance between, difference between, more than and less than, so that pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.

Number – multiplication and division

Statutory requirements

Pupils should be taught to:

- solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Notes and guidance (non-statutory)

Through grouping and sharing small quantities, pupils begin to understand: multiplication and division; doubling numbers and quantities; and finding simple fractions of objects, numbers and quantities.

They make connections between arrays, number patterns, and counting in twos, fives and tens.

Year 2

Number – addition and subtraction

Statutory requirements

Pupils should be taught to:

- solve problems with addition and subtraction:
  - using concrete objects and pictorial representations, including those involving numbers, quantities and measures
  - applying their increasing knowledge of mental and written methods
- recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
- add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
  - a two-digit number and ones
  - a two-digit number and tens
  - two two-digit numbers
  - adding three one-digit numbers
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.

Notes and guidance (non-statutory)

Pupils extend their understanding of the language of addition and subtraction to include sum and difference.

Pupils practise addition and subtraction to 20 to become increasingly fluent in deriving facts such as using 3 + 7 = 10; 10 – 7 = 3 and 7 = 10 – 3 to calculate 30 + 70 = 100; 100 – 70 = 30 and 70 = 100 – 30. They check their calculations, including by adding to check subtraction and adding numbers in a different order to check addition (for example, 5 + 2 + 1 = 1 + 5 + 2 = 1 + 2 + 5). This establishes commutativity and associativity of addition.

Recording addition and subtraction in columns supports place value and prepares for formal written methods with larger numbers.

Number – multiplication and division

Statutory requirements

Pupils should be taught to:

- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Notes and guidance (non-statutory)

Pupils use a variety of language to describe multiplication and division.

Pupils are introduced to the multiplication tables. They practise to become fluent in the 2, 5 and 10 multiplication tables and connect them to each other. They connect the 10 multiplication table to place value, and the 5 multiplication table to the divisions on the clock face. They begin to use other multiplication tables and recall multiplication facts, including using related division facts to perform written and mental calculations.

Pupils work with a range of materials and contexts in which multiplication and division relate to grouping and sharing discrete and continuous quantities, to arrays and to repeated addition. They begin to relate these to fractions and measures (for example, 40 ÷ 2 = 20, 20 is a half of 40). They use commutativity and inverse relations to develop multiplicative reasoning (for example, 4 × 5 = 20 and 20 ÷ 5 = 4).

Year 3

Number – addition and subtraction

Statutory requirements

Pupils should be taught to:

- add and subtract numbers mentally, including:
  - a three-digit number and ones
  - a three-digit number and tens
  - a three-digit number and hundreds
- add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

Notes and guidance (non-statutory)

Pupils practise solving varied addition and subtraction questions. For mental calculations with two-digit numbers, the answers could exceed 100.

Pupils use their understanding of place value and partitioning, and practise using columnar addition and subtraction with increasingly large numbers up to three digits to become fluent (see Mathematics Appendix 1).

Number – multiplication and division

Statutory requirements

Pupils should be taught to:

- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

Notes and guidance (non-statutory)

Pupils continue to practise their mental recall of multiplication tables when they are calculating mathematical statements in order to improve fluency. Through doubling, they connect the 2, 4 and 8 multiplication tables.

Pupils develop efficient mental methods, for example, using commutativity and associativity (for example, $4 \times 12 \times 5 = 4 \times 5 \times 12 = 20 \times 12 = 240$) and multiplication and division facts (for example, using $3 \times 2 = 6$, $6 \div 3 = 2$ and $2 = 6 \div 3$) to derive related facts (for example, $30 \times 2 = 60$, $60 \div 3 = 20$ and $20 = 60 \div 3$).

Pupils develop reliable written methods for multiplication and division, starting with calculations of two-digit numbers by one-digit numbers and progressing to the formal written methods of short multiplication and division.

Pupils solve simple problems in contexts, deciding which of the four operations to use and why. These include measuring and scaling contexts, (for example, four times as high, eight times as long etc.) and correspondence problems in which m objects are connected to n objects (for example, 3 hats and 4 coats, how many different outfits?; 12 sweets shared equally between 4 children; 4 cakes shared equally between 8 children).

Year 4

Number – addition and subtraction

Statutory requirements

Pupils should be taught to:

- add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.

Notes and guidance (non-statutory)

Pupils continue to practise both mental methods and columnar addition and subtraction with increasingly large numbers to aid fluency (see Mathematics Appendix 1).

Number – multiplication and division

Statutory requirements

Pupils should be taught to:

- recall multiplication and division facts for multiplication tables up to 12 × 12
- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

Notes and guidance (non-statutory)

Pupils continue to practise recalling and using multiplication tables and related division facts to aid fluency.

Pupils practise mental methods and extend this to three-digit numbers to derive facts, (for example 600 ÷ 3 = 200 can be derived from 2 x 3 = 6).

Pupils practise to become fluent in the formal written method of short multiplication and short division with exact answers (see Mathematics Appendix 1).

Pupils write statements about the equality of expressions (for example, use the distributive law 39 × 7 = 30 × 7 + 9 × 7 and associative law (2 × 3) × 4 = 2 × (3 × 4)). They combine their knowledge of number facts and rules of arithmetic to solve mental and written calculations for example, 2 x 6 x 5 = 10 x 6 = 60.

Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as the numbers of choices of a meal on a menu, or three cakes shared equally between 10 children.

Year 5

Number – addition and subtraction

Statutory requirements

Pupils should be taught to:

- add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- add and subtract numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

Notes and guidance (non-statutory)

Pupils practise using the formal written methods of columnar addition and subtraction with increasingly large numbers to aid fluency (see Mathematics Appendix 1).

They practise mental calculations with increasingly large numbers to aid fluency (for example, 12 462 – 2300 = 10 162).

Number – multiplication and division

Statutory requirements

Pupils should be taught to:

- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- multiply and divide numbers mentally drawing upon known facts
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
- recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3})
- solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

Notes and guidance (non-statutory)

Pupils practise and extend their use of the formal written methods of short multiplication and short division (see Mathematics Appendix 1). They apply all the multiplication tables and related division facts frequently, commit them to memory and use them confidently to make larger calculations.

They use and understand the terms factor, multiple and prime, square and cube numbers.

Pupils interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (for example, $98 \div 4 = \frac{98}{4} = 24$ r 2 $= 24\frac{1}{2} = 24.5 \approx 25$).

Pupils use multiplication and division as inverses to support the introduction of ratio in year 6, for example, by multiplying and dividing by powers of 10 in scale drawings or by multiplying and dividing by powers of a 1000 in converting between units such as kilometres and metres.

Distributivity can be expressed as $a(b + c) = ab + ac$.

They understand the terms factor, multiple and prime, square and cube numbers and use them to construct equivalence statements (for example, $4 \times 35 = 2 \times 2 \times 35$; $3 \times 270 = 3 \times 3 \times 9 \times 10 = 9^2 \times 10$).

Pupils use and explain the equals sign to indicate equivalence, including in missing number problems (for example, $13 + 24 = 12 + 25$; $33 = 5 \times \square$).

Year 6

Number – addition, subtraction, multiplication and division

Statutory requirements

Pupils should be taught to:

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- perform mental calculations, including with mixed operations and large numbers
- identify common factors, common multiples and prime numbers
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

Notes and guidance (non-statutory)

Pupils practise addition, subtraction, multiplication and division for larger numbers, using the formal written methods of columnar addition and subtraction, short and long multiplication, and short and long division (see Mathematics Appendix 1).

They undertake mental calculations with increasingly large numbers and more complex calculations.

Pupils continue to use all the multiplication tables to calculate mathematical statements in order to maintain their fluency.

Pupils round answers to a specified degree of accuracy, for example, to the nearest 10, 20, 50 etc., but not to a specified number of significant figures.

Pupils explore the order of operations using brackets; for example, 2 + 1 x 3 = 5 and (2 + 1) x 3 = 9.

Common factors can be related to finding equivalent fractions.

Mathematics Appendix 1: Examples of formal written methods for addition, subtraction, multiplication and division

This appendix sets out some examples of formal written methods for all four operations to illustrate the range of methods that could be taught. It is not intended to be an exhaustive list, nor is it intended to show progression in formal written methods. For example, the exact position of intermediate calculations (superscript and subscript digits) will vary depending on the method and format used.

For multiplication, some pupils may include an addition symbol when adding partial products. For division, some pupils may include a subtraction symbol when subtracting multiples of the divisor.

Addition and subtraction

789 + 642 becomes

$$\begin{array}{r} {\scriptstyle 1\ 1\ \ } \\ 7\ 8\ 9 \\ +\ 6\ 4\ 2 \\ \hline 1\ 4\ 3\ 1 \end{array}$$

Answer: 1431

874 – 523 becomes

$$\begin{array}{r} \\ 8\ 7\ 4 \\ -\ 5\ 2\ 3 \\ \hline 3\ 5\ 1 \end{array}$$

Answer: 351

932 – 457 becomes

$$\begin{array}{r} {\scriptstyle 8\ 12\ 1\ \ } \\ \not{9}\ \not{3}\ 2 \\ -\ 4\ 5\ 7 \\ \hline 4\ 7\ 5 \end{array}$$

Answer: 475

932 – 457 becomes

$$\begin{array}{r} {\scriptstyle \ \ 1\ \ 1\ \ } \\ 9\ 3\ 2 \\ -\ \not{4}\ \not{5}\ 7 \\ {\scriptstyle \ \ 5\ \ 6\ \ } \\ \hline 4\ 7\ 5 \end{array}$$

Answer: 475

Short multiplication

24 × 6 becomes

$$\begin{array}{r} {\scriptstyle \ \ 2\ \ } \\ 2\ 4 \\ \times\ \ 6 \\ \hline 1\ 4\ 4 \end{array}$$

Answer: 144

342 × 7 becomes

$$\begin{array}{r} {\scriptstyle 2\ 1\ \ } \\ 3\ 4\ 2 \\ \times\ \ 7 \\ \hline 2\ 3\ 9\ 4 \end{array}$$

Answer: 2394

2741 × 6 becomes

$$\begin{array}{r} {\scriptstyle 4\ 2\ \ \ } \\ 2\ 7\ 4\ 1 \\ \times\ \ \ 6 \\ \hline 1\ 6\ 4\ 4\ 6 \end{array}$$

Answer: 16 446

Long multiplication

24 × 16 becomes

$$\begin{array}{r} {\scriptstyle 2} \\ 24 \\ \times\ 16 \\ \hline 240 \\ 144 \\ \hline 384 \end{array}$$

Answer: 384

124 × 26 becomes

$$\begin{array}{r} {\scriptstyle 1\ 2} \\ 124 \\ \times\ 26 \\ \hline 2480 \\ 744 \\ \hline 3224 \\ {\scriptstyle 1\ 1} \end{array}$$

Answer: 3224

124 × 26 becomes

$$\begin{array}{r} {\scriptstyle 1\ 2} \\ 124 \\ \times\ 26 \\ \hline 744 \\ 2480 \\ \hline 3224 \\ {\scriptstyle 1\ 1} \end{array}$$

Answer: 3224

Short division

98 ÷ 7 becomes

$$\begin{array}{r} 14 \\ {\scriptstyle 2} \\ 7\overline{)\,98} \end{array}$$

Answer: 14

432 ÷ 5 becomes

$$\begin{array}{r} 86\ r2 \\ {\scriptstyle 3} \\ 5\overline{)\,432} \end{array}$$

Answer: 86 remainder 2

496 ÷ 11 becomes

$$\begin{array}{r} 45\ r1 \\ {\scriptstyle 5} \\ 11\overline{)\,496} \end{array}$$

Answer: $45\frac{1}{11}$

Long division

432 ÷ 15 becomes

$$\begin{array}{r} 28\ r12 \\ 15\overline{)\,432} \\ 300 \\ \hline 132 \\ 120 \\ \hline 12 \end{array}$$

Answer: 28 remainder 12

432 ÷ 15 becomes

$$\begin{array}{r} 28 \\ 15\overline{)\,432} \\ 300 \quad 15\times20 \\ \hline 132 \\ 120 \quad 15\times8 \\ \hline 12 \end{array}$$

$\frac{\cancel{12}}{\cancel{15}} = \frac{4}{5}$

Answer: $28\frac{4}{5}$

432 ÷ 15 becomes

$$\begin{array}{r} 28\cdot8 \\ 15\overline{)\,432\cdot0} \\ 30\downarrow \\ \hline 132\downarrow \\ 120\downarrow \\ \hline 120 \\ 120 \\ \hline 0 \end{array}$$

Answer: 28·8